UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2010**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

0883584005

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

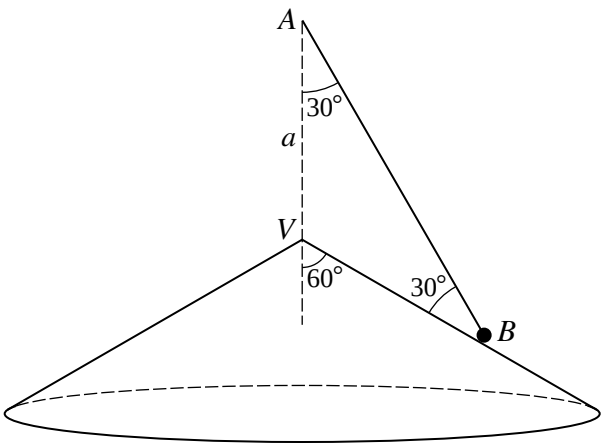
This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 A uniform disc with centre O has mass m and radius a. It is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through O. One end of a light inextensible string is attached to a point on the circumference and is wrapped several times round the circumference. A particle P, of mass $2m$, is attached to the free end of the string and the disc is held at rest with P hanging freely. The system is released from rest. Assuming that resistances may be neglected, find the acceleration of P. [5]

2



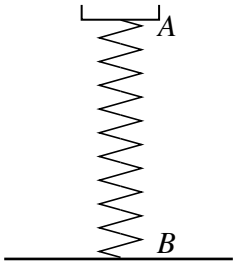
A particle of mass m is attached to the end B of a light inextensible string. The other end of the string is attached to a fixed point A which is at a distance a above the vertex V of a circular cone of semi-vertical angle 60°. The axis of the cone is vertical. The particle moves with constant speed u in a horizontal circle on the smooth surface of the cone. The string makes a constant angle of 30° with the vertical (see diagram). The tension in the string and the magnitude of the normal force acting on the particle are denoted by T and R respectively. Show that

$$T = \frac{m}{\sqrt{3}}\left(g + \frac{2u^2}{a}\right),$$

and find a similar expression for R. [6]

Deduce that $u^2 \leqslant \frac{1}{2}ga$. [1]

3



A spring balance is modelled by a vertical light elastic spring AB, of natural length 0.25 m and modulus of elasticity λ N. The bottom end B of the spring is fixed, and the top end A is attached to a small tray of mass 0.1 kg which is free to move vertically (see diagram). When in the equilibrium position, $AB = 0.24$ m. Show that $\lambda = 25$. [2]

The tray is pushed down by 0.02 m to the point C and released from rest. At time t seconds after release the displacement of the tray from the equilibrium position is x m. Show that

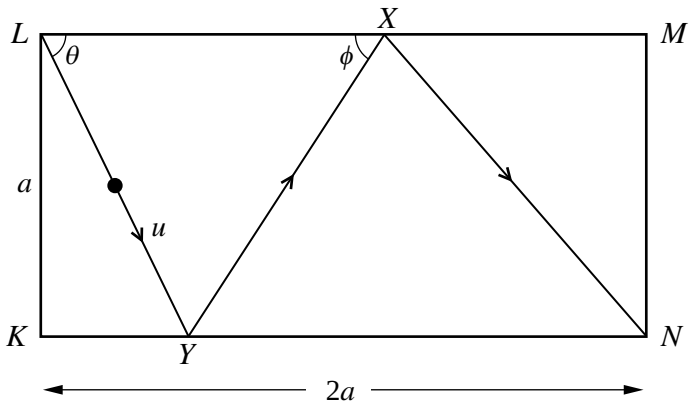
$$\ddot{x} = -1000x. \quad [2]$$

Find the time taken for the tray to move a distance of 0.03 m from C. [5]

4 A small ball P, of mass 40 grams, is dropped from rest at a point A which is 10 m above a fixed horizontal plane. At the same instant an identical ball Q is dropped from rest at the point B, which is vertically below A and at a height of 5 m above the plane. The coefficient of restitution between Q and the plane is $\frac{1}{2}$. Find the magnitude of the impulse exerted on Q by the plane. [4]

The balls collide after Q rebounds from the plane and before Q hits the plane again. Find the height above the plane of the point at which the collision occurs. [6]

5



A rectangular pool table $KLMN$ has $KL = a$ and $KN = 2a$. A ball lies at rest on the table just outside the pocket at L and is projected along the table with speed u in a direction making an angle θ with the edge LM. The ball hits the edge KN at Y, rebounds to hit the edge LM at X and then rebounds into the pocket at N. Angle LXY is denoted by ϕ (see diagram). The coefficient of restitution between the ball and an edge is $\frac{3}{4}$, and all resistances to motion may be neglected. Show that $\tan\phi = \frac{3}{4}\tan\theta$. [3]

Show that $XM = (2 - \frac{7}{3}\cot\theta)a$, and find the value of θ. [6]

Find the speed with which the ball reaches N, giving the answer in the form ku, where k is correct to 3 significant figures. [3]

6 The amount of caffeine in a randomly selected cup of coffee dispensed by a machine has a normal distribution. The amount of caffeine in each of a random sample of 25 cups was measured. The sample mean was 110.4 mg and the unbiased estimate of the population variance was 50.42 mg^2. Calculate a 90% confidence interval for the mean amount of caffeine dispensed. [4]

7 Benford's Law states that, in many tables containing large numbers of numerical values, the probability distribution of the leading non-zero digit D is given by

$$P(D = d) = \log_{10}\left(\frac{d+1}{d}\right), \quad d = 1, 2, \ldots, 9.$$

The following table shows a summary of a random sample of 100 non-zero leading digits taken from a table of cumulative probabilities for the Poisson distribution.

Leading digit	1	2	3	4	5	⩾6
Frequency	22	21	13	11	11	22

Carry out a suitable goodness of fit test at the 10% significance level. [8]

8 A certain mechanical component has a lifetime, T months, which has a negative exponential distribution with mean 2.5.

(a) A machine is fitted with 5 of these components which function independently.

(i) Find the probability that all 5 components are operating 3 months after being fitted. [2]

(ii) Find also the probability that exactly two components fail within one month of being fitted. [3]

(b) Show that the probability that n independent components are all operating c months after being fitted is equal to the probability that a single component is operating nc months after being fitted. [4]

9 **(a)** The following are values of the product moment correlation coefficient between the x and y values of three different large samples of bivariate data. State what each indicates about the appearance of a scatter diagram illustrating the data.

(i) −1,

(ii) 0.02,

(iii) 0.92.

[3]

(b) In 1852 Dr William Farr published data on deaths due to cholera during an outbreak of the disease in London. The table shows the altitude (in feet, above the level of the river Thames) at which people lived and the corresponding number of deaths from cholera per 10 000 people.

Altitude, x	10	30	50	70	90	100	350
Number of deaths, y	102	65	34	27	22	17	8

[$\Sigma x = 700$, $\Sigma x^2 = 149\,000$, $\Sigma y = 275$, $\Sigma y^2 = 17\,351$, $\Sigma xy = 13\,040$.]

(i) Calculate the product moment correlation coefficient. [3]

(ii) Test, at the 5% significance level, whether there is evidence of negative correlation. [4]

10 Carpal Tunnel syndrome is a condition which affects a person's ability to grip with their hands. Researchers tested a treatment for this syndrome which was applied to 8 randomly chosen patients. A pre-treatment and a post-treatment test of grip was given to each patient, with the following results, measured in kg.

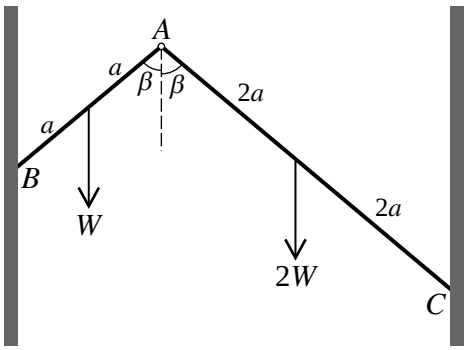
Patient	1	2	3	4	5	6	7	8
Pre-treatment grip	24.3	29.5	28.0	28.5	21.5	28.7	25.1	26.3
Post-treatment grip	28.3	34.6	30.3	31.6	21.5	29.8	26.0	27.5

Stating any required assumption, test, at the 1% significance level, whether the mean grip of people with the syndrome increases after undergoing the treatment. [9]

It is given that there is evidence at the 10% significance level that the mean grip increases by more than w kg. Find an inequality for w. [3]

11 Answer only **one** of the following two alternatives.

EITHER



Two uniform rods AB and AC have lengths $2a$ and $4a$ and weights W and $2W$ respectively. They are freely hinged together at A and rest in equilibrium in a vertical plane with B and C in contact with two rough parallel vertical walls. The plane containing the rods is perpendicular to the walls. The rods AB and AC each make an angle β with the vertical (see diagram). Show that the magnitude of the frictional force acting on AB at B is $\frac{5}{4}W$. [7]

Given that the coefficient of friction at B and at C is μ, find the set of possible values of μ in terms of β. [7]

OR

Aram is a packer at a supermarket checkout and the time he takes to pack a randomly chosen item has mean 1.5 s and standard deviation 0.4 s. Justifying any approximation that you make, find the probability that Aram will pack 50 randomly chosen items in less than 70 s. [4]

Find the greatest number of items that Aram could pack within 70 s with probability at least 90%. [6]

Huldu is also a packer at the supermarket. The time that she takes to pack a randomly chosen item has mean 1.3 s and standard deviation 0.5 s. Aram and Huldu each have 50 items to pack. Find the probability that Huldu takes a shorter time than Aram. [4]

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